The Effect of Vibration on Natural Convective Mass Transfer

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Small horizontal cylinders subliming to room air were vibrated in a vertical direction at 20 to 118 cycles/sec. Increases of up to 660% in the coefficient of mass transfer were thus obtained. The coefficient increased with both frequency and amplitude, the latter having the more pronounced effect. Results are correlated in terms of the stretched vibrational Reynolds number introduced in an earlier study and compared with the analogous case of natural convective heat transfer.

The effect of vibration and pulsation on transport phenomena has been receiving increased attention in recent years. However much of this attention has been directed toward forced convective systems. The work that has been carried out for natural convection has generally been in heat transfer or insonated mass transfer. A search of the literature revealed nothing for the case of a vibrating surface as such in natural convective mass transfer. Accordingly the present study was initiated in an effort to examine this situation and compare it with the analogous case involving heat transfer.

The earliest significant study of this analogous heat transfer problem appears to be that of Martinelli and Boelter (*5*) who obtained up to a fivefold increase in heat transfer coefficient when subjecting a 0.75-in. diameter, electrically heated, cylinder to vertical sinusoidal vibration at frequencies up to 40 cycles /sec. under water. A theoretical solution was obtained for a vertical wall and empirically modified to fit the experimental data. This experimental work was repeated and extended by Boelter and Mason (*1*) who covered three diameters and found even greater improvement in coefficient. Unfortunately only qualitative agreement with the earlier work was obtained.

One of the present authors, Lemlich (*4*), studied the effect of vertical and horizontal vibration in air on natural convective heat transfer from cylindrical wires of 0.0253 to 0.081-in. diameter at frequencies of 39 to 122 cycles/sec. Increases of up to threefold in coefficient were obtained, and a dimensionless correlation was proposed. Also the notion of the vibration stretching the film was suggested, and an alternative correlation based on this idea was offered.

Tsui (*10*) horizontally vibrated a vertical heated plate in natural convection to air and obtained increases in coefficient of up to 24%. In an interferometric study Shine (*9*) also vibrated a vertical plate in air and obtained increases of up to 40%. Kalashnikov and Cherniken (*3*) found greatly increased heat transfer for a heater vibrated at 1.7 to 26.7 cycles/sec. in various liquids.

Considerable improvement has also been reported with the use of sound or ultrasound. However, although similar in some ways, vibrating the fluid with sound is not quite the same as directly vibrating the surface itself. With sound any alteration of the boundary layer comes from without; with surface vibration it initiates from within. Furthermore for most work with sound the displacement amplitude is relatively small, while with a vibrating surface it can easily exceed the diameter of the body. The streaming patterns for the two types of vibratory system are discussed in a recent paper by Fand and Kaye (*2*). Thus at the present time studies with insonation are only capable of limited comparison with studies involving heat or mass transfer from an oscillating surface.

EXPERIMENT

The system selected was that of natural convective sublimation from small horizontal cylinders to quiet room air. Short lengths of aluminum rod or steel wire were coated by momentarily dipping in molten naphthalene or *d*-camphor. After the coating solidified, the end portions were cleaned off and evened with a sharp blade to give a uniform coated length of approximately twenty times the diameter. Spot checking under a microscope showed a smooth coating.

The sample was then freely suspended horizontally in room air for about 10 min. At the conclusion of this preliminary interval the sample diameter was measured in four places with a micrometer accurate to ± 0.005 mm. Any sample for which a measured diameter deviated by more than ± 3% from its mean was discarded. The acceptably uniform cylindrical sample was next quickly weighed to ± 0.1 mg. on a semiautomatic balance. Then it was transferred to a horizontal position suspended by a little cradle from a 0.9-mm. diameter wire stretched about a meter horizontally between a pair of wooden bridges, as shown in Figure 1. One end of the main wire was fixed. Tension was maintained and adjusted by variable weights at the other end. A short distance from one bridge a flat iron plate approximately 1 sq. cm. in area was soldered to the main wire. Just below this plate a stationary electromagnetic coil was positioned. This coil was connected through an amplifier to a variable frequency sinusoidal oscillator. By matching oscillator frequency with a natural frequency of the main wire (obtained by adjusting tension) vertical vibratory motion was imparted to the wire.

Oscillator and amplifier output were always maintained at no more than 50% of rated output in order to minimize any distortion. By temporarily allowing the wire to vibrate at right angles across a uniform magnetic field, picking up the induced e.m.f., amplifying it, and projecting it on the screen of an oscilloscope, the vibratory motion was shown to be essentially sinusoidal.

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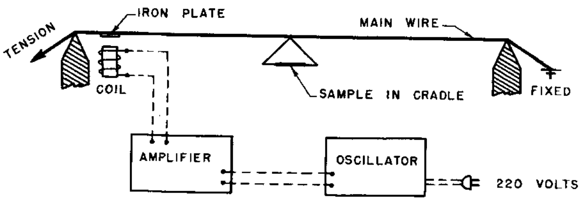

Fig. 1. Schematic diagram.

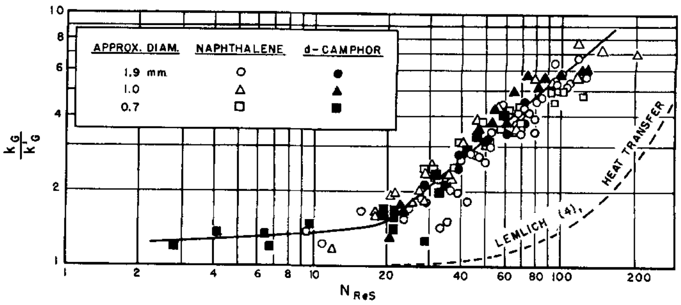

Fig. 3. Correlation of results with stretched vibrational Reynolds number.

The cradle, which consisted of two stiff support wires to hold the sample 4 or 5 cm. below the main wire, was soldered to the main wire at one point near an antinode. By soldering and adjusting carefully no spurious horizontal motion was imparted to the sample, and it followed the vertical motion of the wire. This was checked by viewing the operation under slightly detuned stroboscopic light which gave a slow-motion effect to reveal any undesirable movement. This stroboscope, calibrated with a reed against the 50 cycles/sec. line voltage, was used to measure the frequency of vibration. Amplitude of vibration was measured to ± 0.002 cm. with a traveling telescope.

Duration of a run was such as to permit the sublimation of a few mg. The time required for this varied from about 10 to 100 min. At the conclusion of a run the sample was quickly reweighed. The diameter was remeasured at four places for averaging with initial values. Reduction in diameter over the course of a run usually did not exceed 10%. Any runs where sublimation proceeded down to the metal base were of course discarded.

Wet-and dry-bulb temperatures were determined to ± 0.2°C. with a sling psychrometer. Atmospheric pressure was measured with a barometer. The room itself was small, closed, and unheated.

RESULTS

The effect on k_G of three major variables plus the differing physical properties of the two substances were investigated in over one hundred runs carried out with vibration. The variables included diameter of about 0.7 to 1.9 mm., amplitude of vibration (sometimes referred to by others as *double amplitude* and defined as the distance between extreme positions of the sample center line over the course of 1 cycle) ranging from 0.46 to 7.66 mm., and frequency ranging from 20 to 118 cycles/sec. For naphthalene and *d*-camphor the sublimation pressure at 25°C. was 0.0871 and 0.260 mm. Hg respectively (*6*). The average diffusivity for napthalene is reported (*6*) as 0.0611 sq. cm./sec. and for *d*-camphor was calculated (*8*) as 0.0583 sq. cm./sec. These properties yielded an average N_{Sc} of 2.58 and 2.71 respectively. In addition to the runs with vibration one hundred control runs without vibration were carried out.

Tests for Spurious Influences

To check the absence of any hindrance to the motion of the convection currents some special runs were carried out with extra obstructions temporarily placed midway between the cradled sample and the nearest permanent obstruction (including the main wire itself) in any direction. No significant effect on k_G could be found for these extra obstructions, either with or without vibration.

Calculation based on estimated heat transfer coefficient (*4*) and roughly estimated drag coefficient (*7*) indicated that no appreciable heating of the sample resulted from the vibration. This is confirmed by earlier direct experimental work in heat transfer (*4*).

Sublimation rates were too low to induce any significant cooling. This was checked by calculation and confirmed by experiment with thermometer bulbs coated with the subliming substances. Thus the surface temperature of the sample was essentially that of the surrounding air.

Effect of Vibrational Variables

The effect of vibration is conveniently represented by either the ratio of coefficients k_G/k'_G or the fractional increase in coefficient $k_G/k'_G - 1$. The preparation, diameter, and other nonvibratory conditions of each vibrating sample (which is represented by k_G) was maintained as close as possible to those for the corresponding nonvibrational control (which is represented by k'_G). Thus use of this ratio also makes for some cancellation of error. Furthermore since diameter and diffusivity are the same for vibration as for control, $k_G/k'_G = N_{Sh}/N'_{Sh}$.

Increases in coefficient as high as 660% were obtained. The effects of amplitude and frequency are illustrated in Figure 2 for naphthalene samples of approximately 1.9-mm. diameter. Inspection of this figure reveals an increase in coefficient with an increase in amplitude or frequency. The effect becomes more marked as the magnitude of amplitude or frequency increases. These results follow from the disturbance at the film caused by the vibration and are in qualitative agreement with the three earlier heat transfer studies with vibrating cylinders mentioned above. The in-

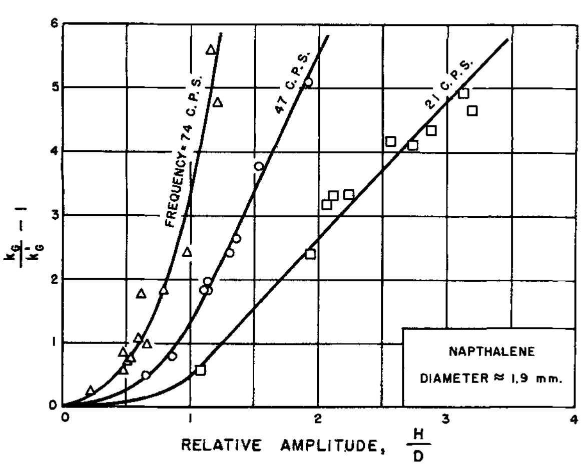

Fig. 2. Typical results with vibrations.

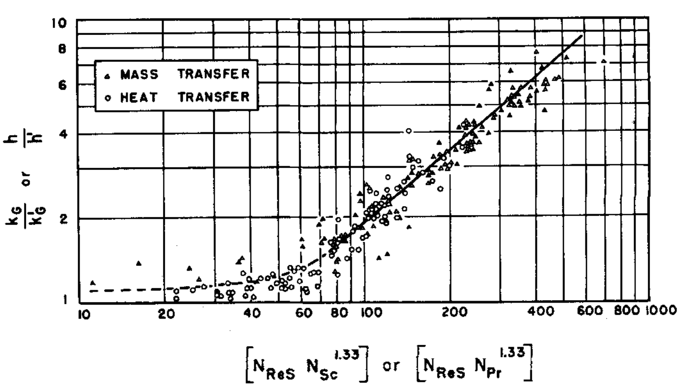

Fig. 4. Correlation of mass transfer results with heat transfer results.

creasingly greater effect of vibration as amplitude or frequency rises is due to the more important role played by the vibration itself at higher levels. For weak vibration the usual natural convection currents control.

As shown in Figure 2 the effect of amplitude appears to be somewhat stronger than the effect of frequency. That this could be so was recognized early by Martinelli and Boelter. However they could not discern such an effect in their experiments, probably because of their relatively low H/D.

DISCUSSION

The greater effect of amplitude as compared with frequency can be made to accord, at least in part, with the approximate stretched film picture mentioned above. The sample cylinder is imagined as vibrating within a well-stirred region surrounded by a relatively stagnant film. The major characteristic linear dimension of this stirred region is the diameter plus the path length, or $D + H$. This can be incorporated with an average velocity $\overline{V} = 2HF$ in a special vibrational Reynolds number defined as follows:

$$N_{ReS} = (D + H)\overline{V}\rho/\mu \qquad (1)$$

N_{ReS} can then be used to correlate the experimental data, as shown in Figure 3. The resulting curve, which shows a break at N_{ReS} of about 20, represents the data within a standard deviation of ±18%. Above the break the curve is drawn straight and is fitted by Equation (2) within a standard deviation of ±19%:

$$\frac{k_G}{k'_G} = 0.117\, N_{ReS}^{0.85} \qquad (2)$$

Alternatively, but with less precision, the trend of the entire curve in Figure 3 may be approximated with a single equation of the same general form as that used for heat transfer in the author's earlier investigation. However such a fit would mask the break.

This corresponding heat transfer correlation from the earlier investigation* is shown in Figure 3. Of course this illustration of a heat transfer correlation on mass transfer coordinates is predicated on the validity of the usual interchange of analogous dimensionless groups, namely N_{Pr} for N_{Sc}, N_{Nu} for N_{Sh} (and therefore h/h' for k_G/k'_G), and $\Delta\rho/\rho$ for $\beta\Delta t$ in N_{Gr}.

By incorporating N_{Sc} or analogously N_{Pr} into the abscissa the data for both investigations can be correlated by one relationship. By restricting consideration to the region above the break (ordinate values above 1.5) this can be accomplished simply. The relationship is shown by the solid straight line in Figure 4 and is represented by Equations (3a) and (3b) within a standard deviation of ±16%:

$$\frac{k_G}{k'_G} = 0.038\, N_{ReS}^{0.85}\, N_{Sc}^{1.13} \qquad (3a)$$

$$\frac{h}{h'} = 0.038\, N_{ReS}^{0.85}\, N_{Pr}^{1.13} \qquad (3b)$$

Of course further work is required to see how well these relationships can be extended beyond the present range of variables.

The break itself is believed to represent a narrow region of profound change in the pattern of vibration-induced streaming currents about the sample. The existence of such a critical region for a vibrating cylinder without heat or mass transfer has been shown experimentally by West (*11*). However the application of West's results as a criterion of the streaming change in the present study would be of doubtful validity at this time because of the unknown interaction between the streaming currents and the natural convective currents.

* A correction should be made in this earlier paper (*4*). The abscissa of Figure 6 in said paper should read 0.1 Re_s and the multiplying constant in the corresponding Equation (5) should be changed from 0.00265 to 0.0000197. The abscissas in the other figures are correct.

SUMMARY OF CONCLUSIONS

1. Vibrating the surface can considerably increase the coefficient of natural convective mass transfer, improvements of up to 660% having been obtained.
2. The coefficient increases with both amplitude and frequency, the influence of the former being the more pronounced.
3. The effect of the vibrational variables can be correlated in terms of the stretched vibrational Reynolds number.
4. Two regions of improvement in coefficient are distinguished, one of low improvement and the other of rapidly rising improvement, with the differences between them attributed to differences in streaming pattern.
5. The improvement in coefficient for the present data is dimensionlessly correlated by Figure 3 or for the upper region by Equation (2).
6. By and large the results are qualitatively in accord with earlier corresponding work in heat transfer. A modification of the correlation to include the heat transfer results of an earlier study by one of the present authors is presented in Figure 4 and for the upper region by Equations (3a) and (3b).

ACKNOWLEDGMENT

This work was supported in part through a fellowship granted by the Technion and indirectly through a Lectureship Grant under the U.S. International Educational Exchange Program (Fulbright Act). It is based in part on the doctoral dissertation of Martin Levy submitted to Technion. Further details may be found in this thesis which is available in English.

The authors also thank Alan Myers for helping with some of the calculations.

NOTATION

D	= diameter of sample, mm.
F	= frequency, cycles/sec.
h	= heat transfer coefficient, cal./(sec.)(sq. cm.)°C.
H	= amplitude, mm.
k_G	= mass transfer coefficient, cm./sec.
N_{Gr}	= Grashof number, dimensionless
N_{Nu}	= Nusselt number, dimensionless
N_{Pr}	= Prandtl number, dimensionless
N_{ReS}	= stretched vibrational Reynolds number, dimensionless
N_{Sh}	= Sherwood number, dimensionless
Δt	= temperature difference, °K.
$\overline{V}$	= average vibrational velocity, mm./sec.
β	= thermal coefficient of volumetric expansion, °K.$^{-1}$
μ	= viscosity, centipoise
ρ	= fluid density, g./(cc.)
$\Delta\rho$	= fluid density difference due to concentration difference between surface and bulk, g./(cc.)

Superscript

′	= no vibration, but same independent nonvibratory variables as with vibration

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Manuscript received September 19, 1960; revision received December 28, 1960; paper accepted January 3, 1961. Paper presented at A.I.Ch.E. New Orleans meeting.